

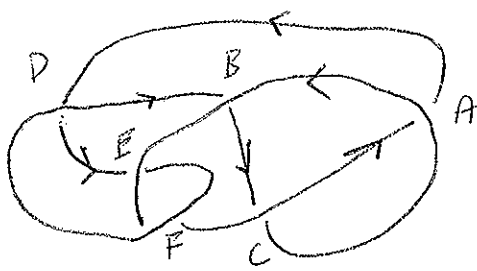
$$W = \sum_{P \in \text{Crossings}} \text{sgn}(P)$$

Solutions to
Coursework #2,
MAS 231 Geometry II
2008

where $\text{sgn} \left(\begin{array}{c} \nearrow \\ \nwarrow \end{array} \right) = +1$

$$\text{sgn} \left(\begin{array}{c} \nearrow \\ \swarrow \end{array} \right) = -1$$

For this choice of flow



(2) for drawing flow
(could be reverse of this)

We have 6 crossings A, B, C, D, E, F.

$$W = -1 + 1 - 1 - 1 + 1 + 1 = 0$$

(3) for each crossing.

(1)

$$\begin{aligned}
 2. \quad B(\infty) &= x^{-1} B(\infty) + x B(0) \quad (3) \\
 &= x^{-1} (1) + x [-(x^2 + x^{-2}) B(0)] \\
 &= x^{-1} + x [-(x^2 + x^{-2}) (1)] \quad (3) \\
 &= x^{-1} - x^3 - x^{-1} = -x^3
 \end{aligned}$$

$$\text{Write } (\infty) = -1 \quad (2)$$

$$\begin{aligned}
 \Rightarrow J(\infty) &= (-x)^{3w} B = (-1)^{3(-1)} (-x^3) \\
 &= -x^{-3} (-x^3) \quad (2) \\
 &= 1.
 \end{aligned}$$

$$B(\bigcirc_P) = x^{-1} B(\infty) + x B(\bigcirc) \quad (3)$$

$$= x^{-1} (x^{-1} B(\infty) + x B(\bigcirc)) \quad (3)$$

$$+ x (x^{-1} B(\bigcirc) + x B(\bigcirc)) \quad (3)$$

$$= x^{-2} B(\infty) (-x^{-2} - x^2) + 2 B(\bigcirc)$$

$$+ x^2 B(\bigcirc) \quad (3)$$

$$= \cancel{(-x^{-4} + 1)} (-x^{-4} + 1) B(\infty)$$

$$+ x^2 B(\bigcirc) \quad (2)$$

$$= (-x^{-4} + 1) (-x^3) + x^2 (-x^3)$$

$$= x^{-7} - x^5 - x^3$$

(3)

~~(3)~~

(above)
(by calculation similar
to those for $B(\infty)$)

$$W\left(\begin{array}{c} \text{---} \\ \text{---} \end{array} \right) = -3$$

~~(2)~~ (2)

$$\text{So } J = (-x)^{3W} B$$

$$= (-x)^{3(-3)} (x^{-7} - x^5 - x^{-3})$$

~~(2)~~ (2)

$$= -x^{-9} (x^{-7} - x^5 - x^{-3})$$

$$= -x^{-16} + x^{-4} + x^{-12}$$

~~(2)~~ (2)

$$= -t^{-4} + t^{-1} + t^{-3}$$

(2)

where $t = x^4$

~~$$B(0,0)$$~~

~~$$= (x^2 + x^4) B(0,0)$$~~

~~$$(0,0)$$~~

~~$$(x^2 + x^4) [x^{-1} B(0,0) + x B(0,0)]$$~~

~~$$(x^2 + x^4) [x^{-1} B(0,0) + x B(0,0)]$$~~

3. $B(\text{c})$

$$= B(0)$$

$$= 1$$

(because bracket is invariant under type II moves)

(7)

(3)

(they may use the definition of Kauffman Bracket)

4. $B(\text{c} \cup \text{c})$

$$= -(x^2 + x^{-2}) B(\text{c})$$

(7)

$$= -(x^2 + x^{-2}) (1)$$

(3)

$$= -(x^2 + x^{-2})$$

5. $J_R(t) = J(t^{-1})$ and $J_R(t) = J(t)$ because the knot is achiral,

IA $J(t) = \sum_{k=-\infty}^{\infty} C_k t^k$

(*) so $J(t) = J(t^{-1})$

(10)

then ~~$J_R(t)$~~ $J(t^{-1})$

$$= \sum_{k=-\infty}^{\infty} C_k (t^{-1})^k$$

$$= \sum_{k=-\infty}^{\infty} C_k t^{-k}$$

$$= \sum_{l=-\infty}^{\infty} C_{-l} t^l$$

where $l = -k$

(**) (10)

Equating coefficients in (*) and (**) gives

$$C_k = C_{-k}$$

(5)